

**INDIAN STATISTICAL INSTITUTE
CHENNAI CENTRE**

M. STAT.(NB-STREAM)-I YEAR

ANALYSIS-I

MID-SEMESTER EXAMINATION-SEPTEMBER 2016

Time: 2 Hours

Marks : 30

Instructions

- Please write your NAME and ROLL NUMBER clearly in the first page.
- Use of mobile phones, tablets, calculators, math. tables, etc., are not permitted.
- Justify/write your answers by clearly stating the appropriate results/theorems that you use whenever required.

Answer all the questions

1. State the following: (4 × 1 = 4 Marks)
 - (a) Archimedian Property of \mathbb{R} .
 - (b) Nested Intervals Property of \mathbb{R} .
 - (c) Cauchy-Schwarz inequality.
 - (d) Bolzano-Weirstrass Theorem in \mathbb{R} .
2. Show that the set of all sequences with values 0 or 1 is uncountable. (2 Marks)
3. Show that the number e is irrational. (2 Marks)
4. Show that the sequence $\{1, \sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots\}$ converges. (2 Marks)
5. Test the series $\sum \frac{(-1)^n}{\ln n}$ for absolute and conditional convergence. (2 Marks)
6. Justify the following: (2 × 1 = 2 Marks)
 - (a) Consider the set of rational numbers \mathbb{Q} with usual addition, multiplication, and ordering. Show that \mathbb{Q} does not satisfy the LUB axiom.
 - (b) Is $\sqrt{2} + \sqrt{3}$ rational?
7. Discuss the convergence of the sequence $\left(\frac{p^n}{1+p^{2n}}\right)$ for various values of p . (3 Marks)
8. Consider the sequence (x_n) of real numbers defined by $x_1 = 1$ and $x_{n+1} = 1 + \left(\frac{1}{1+x_n}\right)$, for $n = 1, 2, \dots$. Show that (x_n) converges and that $\lim(x_n) = \sqrt{2}$. (4 Marks)
9. State and prove Cauchy condensation test. Use this to discuss the convergence of $\sum_{n=3}^{\infty} \frac{1}{n \log n \log \log n}$. (2 + 2 = 4 Marks)
10. Find the lim sup and lim inf of the sequence (x_n) defined by (5 Marks)

$$x_1 = \frac{1}{3}, \quad x_{2n} = \frac{1}{3}x_{2n-1}, \quad \text{and } x_{2n+1} = \frac{1}{3} + x_{2n}, \quad n = 1, 2, \dots$$